THE INTERNATIONAL JOURNAL OF SCIENCE & TECHNOLEDGE

Hamiltonian Disemigraphs

Hamida Aktara Hoque
Research Scholar, Department of Mathematics, Gauhati University, Guwahati, Assam, India
P. Das
Associate Professor (Retired), Gauhati University, Guwahati, Assam, India

Abstract:
The concept of Eulerian and Hamiltonian graphs was extended by E. Sampath kumar to semigraph in [2]. He also introduced the idea of Eulerian Disemigraphs and obtained few characterizations. Here we have focused our attention to obtain conditions for a Disemigraph to be Hamiltonian.

Keywords: *Semigraph, disemigraph, hamiltonian disemigraph.*

1. Introduction

Hamiltonian paths and cycles are named after Sir William Rowan Hamilton who invented the puzzle that involves the problem of finding a hamilton cycle in the edge graph of a dodecahedron in 1859.Looking at the structure of a disemigraph vis a vis certain real life situations, the possibility of application of hamiltonian disemigraph in fields like permutation, combination, scheduling, sitting arrangement etc. cannot be ruled out. In [2], E. Sampathkumar introduced the concept of Eulerian as well as Hamiltonian semigrahs and derived some significant characterizations regarding them. He also extended the eulerian concept to disemigraph setting, but the concept of a hamiltonian disemigraph was still missing in the literature. Here, we have attempted to introduce the idea of a hamiltonian disemigraph and in the process, obtained various types of hamiltonian disemigraphs which are analogous to those in digraphs and semigraphs. In this regard we are successful in deriving few characterizations of a hamiltonian disemigraph.

2. Preliminaries

For terminology and definitions not defined here we refer the readers to [2],[3]and[17].

A *graph*G (V, X) consists of a finite nonempty set V of points together with a prescribed set X of unordered pairs of distinct points of V.

A *digraph*D (V, A) consists of a set V of vertices and a set A of arcs (directed edges).

A digraph D of order $n\geq 3$ is called *pancyclic*if D contains cycles of every length l for $3\leq l\leq n$. A digraph D is said to be *regular* of degree k (or k-regular) if each vertex$v$ of D dominates k vertices and is itself dominated by k vertices. A digraph D is said to be*hamiltonian* if it contains a cycle containing all the vertices of D and such a cycle is called a *hamilton cycle*. A digraph D is *hamiltonian-connected* if D contains a hamiltonian u-v path for any two distinct vertices u and v of D.

A *semigraph*G is a pair (V, E), where V is a non-empty set whose elements are called vertices of G, and E is a set of n-tuples, called edges of G of distinct vertices, for various $n\geq 2$ satisfying the following conditions-

S.G.1- Any two edges have at most one vertex in common.

S.G.2- Two edges (u_1, u_2,,u_n) and (v_1,v_2,…,v_m) are considered to be equal if and only if

(i)$m=n$ and

(ii) either $u_i=v_i$ for $1\leq i\leq n$, or $u_i=v_{n-i+1}$ for $1\leq i\leq n$.

All vertices on an edge of a semigraph are considered to be adjacent to one another.

A Semigraph G is *hamiltonian* if G contains a cycle C containing all the vertices of G, and C is called a *hamilton cycle*. A Semigraph G is *s-hamiltonian* if it has a hamilton s-cycle, and G is *w-hamiltonian* if it has a hamilton w-cycle.

A *disemigraph* or directed semigraph D is a finite set of objects called vertices together with a (possibly empty) set of ordered n-tuples of distinct vertices of D for various $n\geq 2$, called directed edges or arcs, satisfying the following condition-

"For any two distinct vertices u and v in a disemigraph D, there is at most one arc containing u and v such that u is adjacent to v and at most one arc containing u and$v$ such that v is adjacent to u".

While drawing a disemigraph in a plane, the initial (or terminal) vertex of an arc which is not a middle vertex of any arc is represented by a thick dot and the middle vertices of an arc are represented by small circles. If a middle vertex is also an initial (or terminal) vertex of an arc, we draw a small tangent to the circle.

If $a=(u_1,u_2,\ldots,u_n)$ is an arc in a disemigraph *D* then a *subarc* of *a* is an *r*-tuple $(u_{i_1}, u_{i_2}, \ldots u_{i_r})$ where $1\le i_1<i_2<\ldots<i_r\le n$. A *partial arc* of *a* is a $(j-i+1)$-tuple $(u_i,u_j)=(u_i,u_{i+1},\ldots,u_j)$ where $1\le i<j\le n$. We say that u_i is adjacent to all other u_j, $1\le i<j\le n$ and u_j is adjacent from u_i, $1\le i<j\le n$. The out-degree (in-degree) of a vertex *u* of *D* is the number of vertices of *D* that are adjacent from (adjacent to) *u*. The *degreedeg*(u) or $d(u)$ of *u* is defined to be the sum of the in-degree and the out-degree of *u*. That is, $d(u)=od(u)+id(u)$.The concepts of sub arc and partial arc lead to two different types of paths. A path *P* in a disemigraph *D* is called *s-path* (strong path) if any two consecutive vertices on it are also consecutive vertices of an arc in *D* otherwise *P* is called a *w-path* (weak path).A*semipath* connecting two vertices u_o and u_n in a disemigraph *D* is a finite sequence of distinct vertices $u_ou_1u_2\ldots u_n$in *D* such that for each *i*, $0\le i\le n-1$, either u_i is adjacent to u_{i+1} or u_{i+1} is adjacent to u_i.
An *s-cycle(w-cycle)* in *D* is a closed *s*-path(*w*-path) in *D*.
The *adjacency digraph* D_a of a disemigraph *D* has $V(D)$ as vertex set where for any two vertices *u* and *v*, *u* is adjacent to *v* if and only if it is so in *D*. For any vertex *v* in *D*, $id(v)$ respectively ($od(v)$) in *D* is the same as $id(v)$ respectively ($od(v)$) in D_a. The distance between any two vertices in *D* is the distance between them in the underlying adjacency digraph D_a of *D*.
The *consecutive adjacency digraph*D_{ca} of *D* has $V(D)$ as its vertex set where for any two vertices *u* and *v*, *u* is adjacent to *v* if and only if,
(i) *u* is adjacent to *v* and
(ii) *u* and *v* are consecutive vertices of an arc in *D*.
A disemigraph *D* is *simple* if any two arcs in *D* either contain atmost one vertex or all vertices in common. A disemigraph *D* is *oriented* if *D* contains no symmetric pair of arcs.
There are manyresults for hamiltonian graphs in [10],[12],[17] etc., very few of which are known to guarantee the hamiltonity in digraphs. One has to go through specific results for hamiltonian property concerning directions in [1],[4],[9],[17],[18],[19],[20] etc.,most of which usually take the form involving many arcs and so not applicable to oriented graphs. The early results on sufficient conditions for hamiltonian digraphs are due to Ghouila-Houri [1], Manoussakis [9] and Woodall[19]. A more general result is due to Meyniel[20].
No elegant characterization of hamiltonian graphs or digraphs exists, although several necessary and sufficient conditions are known. Here, we reproduce some results for digraphs-
Theorem [1]: If *D* is a strongly connected digraph of order *n* and if $id(v)\ge n/2$ and $od(v)\ge n/2$ for all vertices *v* in *D*, then *D* has a hamilton cycle.
Theorem [1]: If *D* is a strongly connected digraph of order *n* and if $d(v)\ge n$ for all vertices *v* in *D*, then *D* is hamiltonian.
Theorem [1]: If *D* is strongly connected digraph of order *n* and if $id(v)+od(v)\ge n$ for every vertex *v*, then *D* has a hamilton cycle.
A particular case of the above theorem occurs when *D* is a simple connected digraph of order n.
Theorem [9],[5]: If *D* is a simple connected digraph of order *n* and if for each vertex *v*, $id(v)\ge n/2$ and $od(v)\ge n/2$, then *D* has a hamilton cycle.
Theorem [19]: If *D* is strongly connected digraph of order *n* and if $id(u)+od(v)\ge n$ for all pairs of non-adjacent vertices *u* and *v*, then *D* is hamiltonian.
Theorem [20]: If *D* is strongly connected digraph of order *n* and if $d(u)+d(v)\ge 2n-1$ for all pairs of non-adjacent vertices*u* and *v* in *D*, then *D* is hamiltonian.
Unfortunately, no general result is known for the digraph counterpart.
In this paper we consider only simple and connected disemigraph with at least three arcs unless otherwise stated.

3. Hamiltonian Disemigraphs

There are numerous problems such as mail box collection, school bus scheduling, electricity supply network design, service vehicle routing etc. whichcan be studied in disemigraph setting and solved through the application of hamiltonian properties.
Now we propose few definitions in connections with disemigraphs and derive some of their characterizations.

3.1. Definitions

Analogous to digraphs, we can have different types of connectedness indisemigraphs as follows-
A disemigraph *D* is said to be strong, if any two of its vertices are mutually reachable.
A disemigraph *D* is said to be unilateral, if for any two vertices in *D* at least one of them is reachable from the other.
A disemigraph *D* is said to be weak, if any two of its vertices are joined by a semipath.
A disemigraph *D* is said to be *k*-regularor regular of degree *k* if for any vertex *v*of *D*$id(v)=od(v)=k$. Here we consider only the adjacency degree of a vertex unless stated otherwise.
A disemigraph *D* of order *p* is said to be pancyclic if *D* contains cycles of every length *l* for $3\le l\le p$. If *D* has only *s*-cycles then *D* is said to be*s*-pancyclic otherwise,*D* is called *w*-pancyclic.

3.2. Definitions

A path in a disemigraph*D* containing all the vertices of *D* is said to be a hamilton path.A closed hamilton path is often called a hamilton cycle. Thus a hamilton cycle contains all the vertices of *D* exactly once.
A disemigraph *D* is said to be*s*-hamiltonian(or *w*-hamiltonian) if it contains ahamilton*s*-cycle (or *w*-cycle).
A disemigraph *D* is said to be*p*-hamiltonian(or perfect hamiltonian)if *D* contains a hamilton cyclethat traverses all the arcs of *D*.

Forhamiltonian properties to be satisfied, a disemigraph*D* must be connected with atleast 3 (three) arcs. As every *s*-cycle is a *w*-cycle, so every *s*-hamiltonian disemigraph is a *w*-hamiltonian disemigraph, but the converse may not hold good. However, a *p*-hamiltonian disemigraph is both*s*-hamiltonian and *w*-hamiltonian. A*p*-hamiltonian property of *D* reflects that *D* has a hamilton cycle (possibly *s*-cycle) which traverses all the arcs of *D* exactly once.

Few selected illustrations of the preceding definitions are in order now.

→ Examples 3.1:

i) The following disemigraph is *s*-hamiltonian and therefore,*w*-hamiltonian also but not *p*-hamiltonian. Here (*abcda*) is a hamilton *s*-cycle.

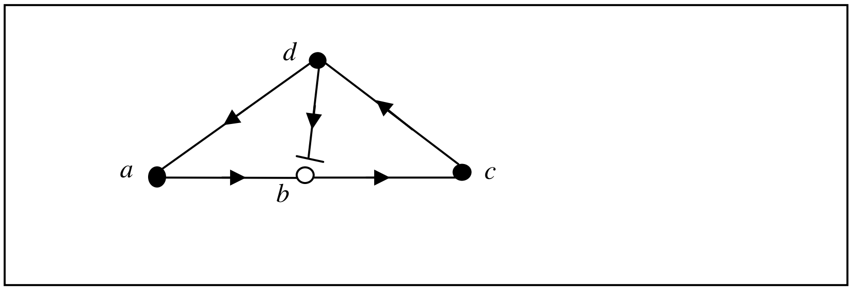

Figure 1

ii) The following disemigraph is *w*-hamiltonian but neither *s*-hamiltonian nor *p*-hamiltonian. Here (*abcdea*) is a hamilton *w*-cycle.

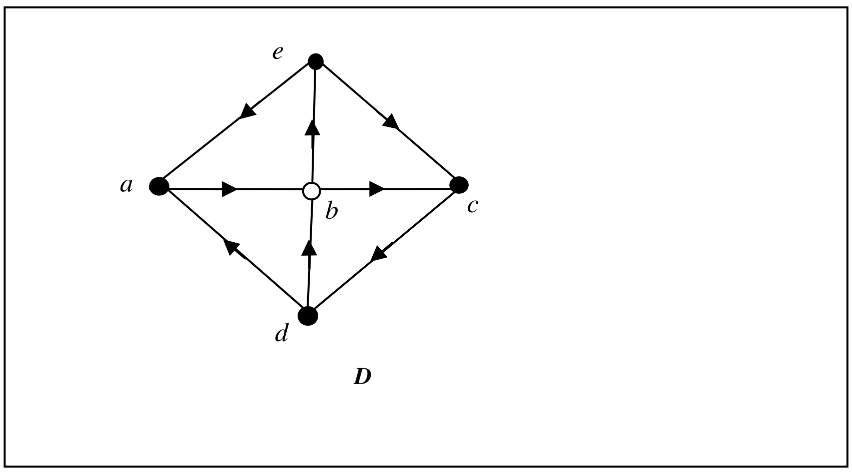

Figure 2

iii) The following disemigraph is *p*-hamiltonian. It is also an *s*- as well as a*w*-hamiltonian disemigraph.

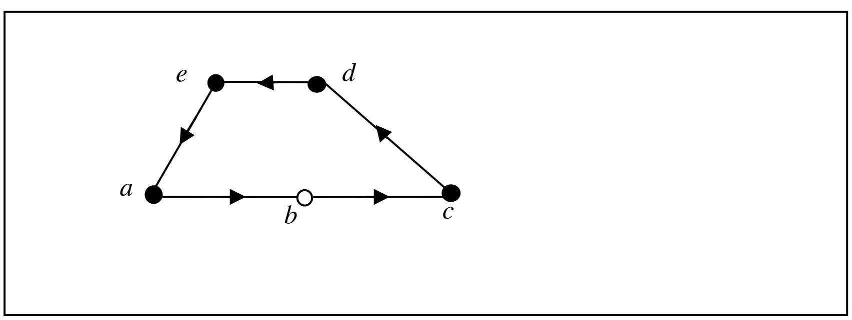

Figure 3

iv) The following disemigraph is not hamiltonian of any kind i.e. non-hamiltonian.

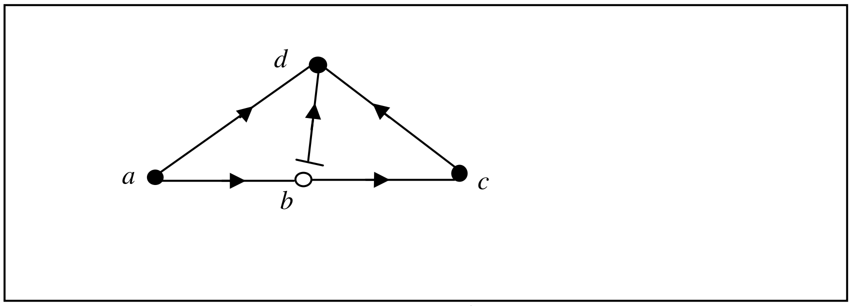

Figure 4

From above definitions we observe that-

i. A disemigraph having a source or sink cannotbe hamiltonian but it mayhavehamiltonian path.
ii. A strong disemigraph as well as a strong and complete disemigraph are both*w*-hamiltonian but they may not be *s*- or *p*-hamiltonian. For example, see Fig.2, where *D* is strong and complete but neither*s*- nor*p*- hamiltonian.
iii. A tournament may not be hamiltonian of any kind but it always contains a hamiltonian *w*-path.
iv. No tournament can be *p*-hamiltonian.

E. Sampathkumar has established some significant results regarding hamiltonian semigraphs in[2] few of which are mentioned in the following.

Proposition 8.7: [2] Any complete semigraph *G* with at least three edges is *s*-hamiltonian.

A disemigraph being complete does not imply to have a hamilton cycle [Fig.4].

Proposition 8.8: [2] If *G* is a semigraph of order $p\geq 3$ such that for all distinct non-adjacent vertices *u* and *v*, $deg_a u+deg_a v\geq p$, then *G* is *w*-hamiltonian.

Proposition 8.10: [2] If *G* is a semigraph of order $p\geq 3$ such that for all distinct non-adjacent vertices *u* and *v*, $deg_{ca}u+deg_{ca}v\geq p$, then *G* is *s*-hamiltonian.

Unfortunately, the foregoing degree related conditions are not generally applicable to disemigraphs.We cite few cases of such observations in the following.

→ Examples3.2:

i) The following disemigraph satisfies the condition $deg_a u+deg_a v\geq p$for any two distinct non-adjacent vertices*u* and *v* but it has no any hamilton cycle (here *p*=5).

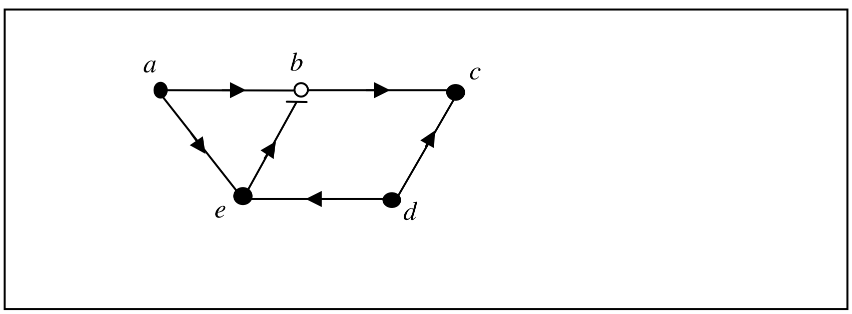

Figure 5

ii) For any two distinct non-adjacent vertices *u* and *v*, the following disemigraph *D* satisfies$deg_{ca}u+deg_{ca}v\geq p$, but *D* does not have evena *w*-hamiltonian path (here *p*=7).

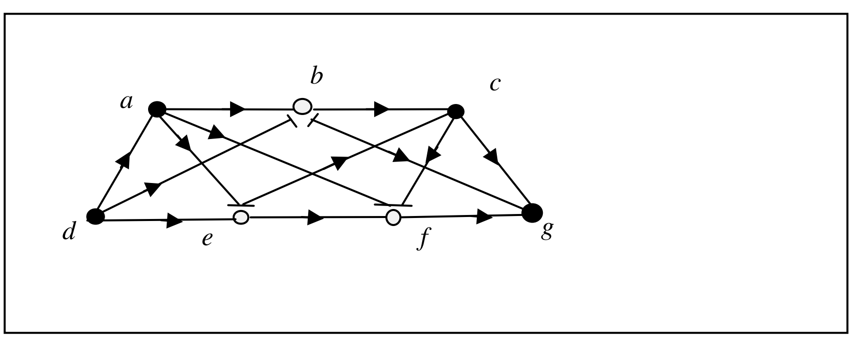

Figure 6

In the following we derive some conditions for disemigraphs to be hamiltonian in different ways.

- Proposition 3.1:A disemigraph$D$ is w-hamiltonian if and only ifD_a is hamiltonian.
- → Proof: By the definition of $D_a$any cycle in D is a cycle in D_a. So D being w-hamiltonian, so isD_a. Conversely, when D_a is hamiltonian then $D_a$contains a hamilton cycle which guarantees a hamilton cycle (which is also aw-cycle) in D. Thus D is w-hamiltonian.
- Corollary 3.1:D is w-hamiltonian, if D_{ca} is hamiltonian but, the converse may not be true.

e.g. In the following D is w-hamiltonian but D_{ca} is not.

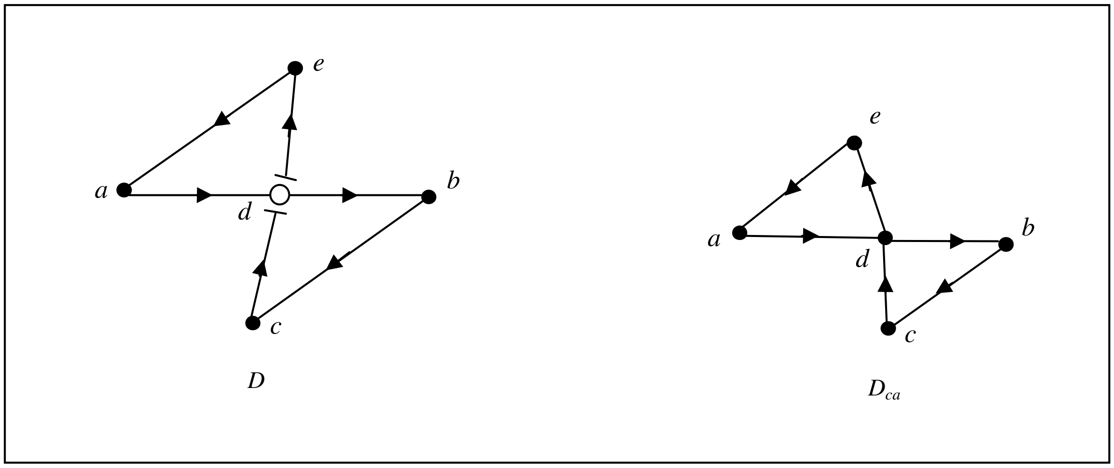

Figure 7

- Proposition 3.2: If a disemigraph D is s-hamiltonian then both$D_a$and D_{ca} are hamiltonian.
- → Proof: Trivial.
- Corollary 3.2: Neither $D_a$nor D_{ca} being hamiltonian implies that D is s-hamiltonian.Fig.2 may be referred.
- Proposition 3.3: If D is a complete disemigraph with $id(v)\geq 1$and $od(v)\geq 1$ for all vertices v in D, then D is w-hamiltonian.
- → Proof:If D is complete and$id(v)\geq 1$ and$od(v)\geq 1$ for all vertices v in D, then D is complete and strong. In this case,D contains a spanning cycle, which is a hamilton w-cycle. Hence D is w-hamiltonian.
- Corollary 3.3: A strong tournament is always w-hamiltonian.
- Proposition 3.4: Let $D(V,E)$ be a disemigraph of order n such that $d(v)\geq n$ for all vertices$v$ in D, then D is w-hamiltonian.
- → Proof:Let$d(v)\geq n$ for all vertices v in D.

<u>*Case i.*</u>v is adjacent with all other $n-1$ vertices.

Then v is incident with atleast one symmetric pair of arcs. D is complete and $id(v),od(v)\geq 1$. By Proposition 3.3, we are done.

<u>*Case ii.*</u>$v(=v_j)$ is not adjacent with at least one v_i for$1\leq i,j\leq n$, say, v_r .

Since $d(v)\geq 1$ for all v in D, there is at least two symmetric pair of arcs in D associated with v_i that contribute degrees to thev_i such that $d(v_i)\geq 1$, $1\leq i\leq n$ .... (1)

If possible, let$D$ be not w-hamiltonian. Then without loss of generality we can assume that $C=(v_1,v_2,...,v_{r-1},v_{r+1},... v_n,v_1)$ is a cycle in D such that v_r is not in C.Then there cannot exist any symmetric pair of arcs in between v_r and any other v_i, $1\leq i\leq n$, otherwise C will contain v_r, which is a contradiction to (1). Thus C contains v_r and so C is a hamilton w-cycle for$D$. So our assumption was wrong. Hence$D$ is w-hamiltonian.

- Corollary 3.4: Every disemigraph with$id(v)\geq n/2$ and $od(v)\geq n/2$, is w-hamiltonian.
- Proposition 3.5: A disemigraph D is s-hamiltonian if D is s-pancyclic.
- → Proof: Let D be of order n and also let D be s-pancyclic, then D has s-cycles of every length$l$ for $3\leq l\leq n$.

The s-cycle of length $l(=n)$ constitutes a hamilton s-cycle for D. Thus D is s-hamiltonian.

- Corollary 3.5: Every pancyclic disemigraph is w-hamiltonian.
- Proposition 3.6: If D is strongly complete and $d(v)\geq n$ for any vertex v in D, then D has a hamiltonian s-path.
- → Proof:Given $d(v)\geq n$ for any vertex v in D, so D is w-hamiltonian (by Proposition 3.4). Let C be a hamilton w-cycle in D. Now,Let us construct a disemigraph D_1 from D by adding a new vertex $h$and joiningit with every vertex of $D$by arcs in both directions.D_1 is also strongly complete and necessarily we can have a hamilton s-cycle, say, C_1, where $C_1=C\cup\{h\}$ and thus D_1 is s-hamiltonian. Now deleting h and its incident arcs in D_1, C_1 produces a hamilton s-path in D. Thus D has a hamilton s-path whenever D is strongly complete and $d(v)\geq n$ for any vertex v in D.
- Corollary 3.6: If D is strongly complete with order n such that $od(v)\geq n/2$ and $id(v)\geq n/2$ for every vertex v in D then D has a hamilton s-path.
- Proposition3.7:D is p-hamiltonian, if and only if,D itself is a hamilton cycle.
- → Proof:Let D be$p$-hamiltonian.Then D contains a hamilton cycle Cthat traverses all the arcs of D. If possible, let D be not a hamilton cycle,then D will contain at least one arc (u,v) such that $D-(u,v)$ is a hamilton cycle which contradicts the fact that$D$

has a hamilton cycle C traversing all the arcs of D. Hence D itself is the hamilton cycle. Conversely, if D itself is a hamilton cycle then obviously D is a cycle containing all the vertices of D and traversing all arcs of D. Thus D is p-hamiltonian.

- Corollary 3.7: Only spanning cycles are p-hamiltonian.
- Corollary 3.8: D is p-hamiltonian, if and only if, D_{ca} is 1-regular.

➢ Proposition 3.8: If D is a p-hamiltonian disemigraph then it is both s-hamiltonian and w-hamiltonian.

→ Proof: D being p-hamiltonian, D contains a hamilton cycle C that traverses all the arcs of D. By Proposition 3.7. D itself is the hamilton cycle C i.e. $D=C$. D is the only cycle which is obviously the hamilton s-cycle and so is the hamilton w-cycle. Thus D is with both hamilton s-cycle and hamilton w-cycle. Hence D is both an s-hamiltonian and a w-hamiltonian whenever D is a p-hamiltonian disemigraph.

In the following we cite an example as an application of hamilton cycle.

Application: In a chemical factory, batches of drugs D_1, D_2,....., D_n are manufactured in a single reaction vessel, one at a time. If D_j is to follow D_i and the vessel has to be cleaned, at a cost c_{ij}. The batches are to be manufactured in a continuous and cyclic manner, so that once the last batch has been produced, the first batch has to begin again. The problem (scheduling problem) is to find the production sequence with the least cleaning cost. This situation can be represented by a disemigraph and then the problem is to find a hamilton cycle.

3.3. Definitions

A disemigraph D is said to be hamiltonian s-connected if for any two distinct vertices u and v of D, D contains a hamilton s-path in between u and v.

A disemigraph D is said to be hamiltonian w-connected if D contains a hamilton w-path in between any two distinct vertices u and v of D.

From the above definitions it follows immediately that

i) A hamiltonian disemigraph may not be hamiltonian connected.

ii) If D is hamiltonian s-connected then D is both s- and w-hamiltonian.

iii) If D is hamiltonian w-connected then D is w-hamiltonian.

Now, we give a characterization of hamiltonian w-connected disemigraph involving condition on degrees of its vertices.

➢ Proposition 3.9: Let D be a disemigraph of order n with $od(u)+id(v)\geq n+1$ for any two distinct non adjacent vertices u and v of D. Then D is hamiltonian w-connected.

→ Proof: Let x and y be any two distinct vertices in D. Let us construct a disemigraph D_1 from D by adding a new vertex h and joining it with every vertex of D by arcs in both directions. Here D_1 becomes a strong disemigraph of order $n+1$ and such that $od(z)\geq n+1$, $id(z)\geq n+1$ for any z in D_1. We claim that D_1 is w-hamiltonian. Now, if x and y are adjacent in D_1 then D_1 is complete and also in D_1, $od(z)\geq n+1$ and $id(z)\geq n+1$. Then by Proposition 3.3, D_1 is w-hamiltonian. Again if x and y are non adjacent then $od(x)+id(y)\geq(n+1)+1=n+2$. If we can show $d(z)\geq n+1$ for any z in D_1, then by Proposition 3.4 D_1 is w-hamiltonian. If possible, let $d(z)<n+1$, for any z in D_1.

Now $od(x)+id(y)\geq n+2$

$\Rightarrow d(x)+d(y)\geq 2(n+2)$

$\Rightarrow d(x)\geq 2(n+2)-(n+1)$, ($\because d(y)<n+1$, for any y in D_1)

$\Rightarrow d(x)\geq n+3$, for any x in D_1, which is a contradiction to the fact that $d(x)<n+1$, for any x in D_1.

So D_1 is w-hamiltonian and thus it contains a hamilton w-cycle C (say). Deleting h and its incident arcs in D_1, C produces a hamiltonian w-path P (say). We claim that P is the required hamiltonian x-y path. Let us go back to D_1. If x and y are adjacent in D_1 then D_1 being complete, strong and w-hamiltonian we can construct C as $C=(...yhx...)$; if x and y are non adjacent in D_1 then (y,h) and (h,x) are in D_1. So, we can have C as $C=(...yhx...)$. Thus for any two vertices x and y in D_1 we are left with an x-y path $P=C-\{h\}$ as the hamilton w-path in D. Thus, D is hamiltonian w-connected.

Here, we have found only a few partial characterizations for hamiltonian disemigraphs. More elegant characterization of a hamiltonian disemigraph is still an open problem.

4. References

i. Ghouila-Houri, A. (1960). Une condition suffisante d′existence d′um circuit hamiltonian, C.R.Acad.Sci. Paris 25, 495-497.

ii. Sampathkumar, E. (2000). Semigraphs and Their Applications, Report on the DSTProject, Report No. DST/MS/022/94, Govt. of India.

iii. Harary, F.(2001). Graph Theory, Narosa Publishing House, Reprint.

iv. Gutin, G. &yeo, Anders. (2001) Remarks on hamiltonias digraphs, Australian Journal of Combinatorics 23, 115-118.

v. Paul, M. J., Shershin, C. B. & Shershin, A.C. (1991). Note on sufficient conditions for a graph to be hamiltonian , International J.Math.&Math. Sci. Vol. no.4, 825-827.

vi. Rahman, M. S., Kaykobad, M. & Firoz, J. S. (2014).New Sufficient conditions for Hamiltonian Paths, Scientific World Journal.

vii. Ore O., (1960). Note on Hamilton circuits, Amer. Math. Monthly, 67.

viii. Rahman, M. S. &Kaykobad, M. (2005). On Hamiltonian cycles and Hamiltonian Paths. Information processing letters, 94(1):37-41.

ix. Manoussakis, Y. (1992). Directed Hamiltonian graphs, J.Graph Theory 16, 51-59.

x. Chartrand, G. (1985). Introductory Graph Theory, Dover.
xi. Berge, C. (1973).Graphs and Hypergraphs, North Holland, Amsterdam.
xii. Dirac, G. A. (1952). Some theorems on abstract graph, In proceedings of the London Mathematical Society, Third series, Vol.2, pp69-81.
xiii. Das, P. & Hoque, Hamida Aktara. (2012).A Note on Tournaments in Disemigraph, Int. J. Math. Sc. & Engg. Appls. Vol.6, No.II, pp. 299-307.
xiv. Parthasarathy, K. R. Basic Graph Theory, Tata McGraw-Hill Publishing Company Limited, New Delhi.
xv. Sampathkumar, E. (2003). Semigraphs, SERC research Highlights, Department of Science and Technology, New Delhi-16, 370-389.
xvi. Bang-Jensen, Jorgen. & Gutin, Gregory (2007). Digraphs-Theory, Algorithms and Applications, Springer-Verlag.
xvii. Chartrand, G. & Lesnaik, Linda. (1986). Graphs and Digraphs, second edition.
xviii. Harary, F., Norman, R. &Cartwright, D. (1965). Structural Model: An Introduction to the theory of Directed Graphs, Wiley.
xix. Woodall, D.R. (1972). Sufficient conditions for cycle in digraphs, Proc. London Math. Soc.24, 739-755.
xx. Meyniel, H. (1973).Une condition suffisante d'existence d'un circuit hamiltonien dans un graphe oriente′, J.Combin. Theory. Ser.B14, 137-147.
xxi. Al-azemi, Abdullah. (2012). Classification of regular digraphs, normally regular digraphs and strongly regular digraphs, International Journal of Pure and Applied Mathematics, Vol.78, No.3,379-393.