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Multiplication of Numbers Made Easy

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Abstract:
A number is a mathematical object used to count, label, and measure. In mathematics, the definition of number has been extended over the years to include such numbers as 0, negative numbers, rational numbers, irrational numbers, and complex numbers.
Mathematical operations are certain procedures that take one or more numbers as input and produce a number as output. Unary operations take a single input number and produce a single output number. For example, the successor operation adds 1 to an integer, thus the successor of 4 is 5. Binary operations take two input numbers and produce a single output number. Examples of binary operations include addition, subtraction, multiplication, division, and exponentiation. The study of numerical operations is called arithmetic.

Recall that $6 + 6 + 6 = 6 \times 3$
Instead of adding 6 three times, you can multiply 6 by 3 and get 18, the same answer.

Similarly,
$6 + 6 + 6 + 6 + 6 + 6 + 6 = 6 \times 7 = 42$
Still by the same token,
$2 + 2 + 2 + 2 = 2 \times 4$
In algebra, 2×4 can be written as (2)(4)

You can think of this as four groups of 2
Multiplication of numbers has existed as long as numbers existed. Over time people have adopting different strategies of multiplying numbers. There is the known method which we use on daily basis. We shall demonstrate the known method of multiplication of numbers and analyze the number of steps taken to achieve the product. Then we shall demonstrate the new method, analyze it also and see the number of steps taken to get the product.
Then a comparison will be done on which of the two methods is more efficient, in terms of time and memory space for us to perform the computation.
Suppose we want to multiply two numbers each with two values

			5	6
*			5	6
	2	8	0	
+		3	3	6
=	3	1	3	6

Table 1

5*6=30(1)	30/10=3,0(2)	0 carry 3
5*5=25(3)	25+3=28(4)	280(5)
6*6=36(6)	36/10=3,6(7)	6 carry 3
6*5=30(8)	30+3=33(9)	336(10)
2800+336=3136(11)		

Table 2

From the above it is clear that we will require 9 steps to multiply two numbers. Suppose that we wanted to multiply three numbers

					3	9	5
*					4	6	7
		1	5	8	0		
+			2	3	7	0	
=				2	7	6	5
		1	8	4	4	6	5

Table 3

4*5=20(1)	20/10=2,0(2)	0 carry 2
4*9=36(3)	36+2=38(4)	38/10=3,8(5)
8 carry 3	4*3=12(6)	12+3=15(7)
6*5=30(8)	30/10=3,0(9)	0 carry 3
6*9=54(10)	54+3=57(11)	57/10=5,7(12)
7 carry 5	6*3=18(13)	18+5=23(14)
7*5=35(15)	35/10=3,5(16)	5 carry 3
7*9=63(16)	63+3=66(17)	66/10=6,6(18)
6 carry 6	7*3=21(19)	21+6=27(20)
158000+23700(21)	+2765=184465(22)	

Table 4

From the example above it is clear that the steps of multiplication of numbers are many. Considering the fact that computer operates in logical way, then for a short multiplication it would require many steps of computation. This means more processor time and memory space occupied.

After thinking through I have developed a new and a shorter method of number multiplication. This method reduces the number of steps that would be taken before the final answer. The method has special characteristics for numbers with their one's values adding up to ten, squares, and numbers with values that are common with the same powers of 10.

The method applies for any values that have to multiply.

Consider a two value number AB and another two value number CD

$AB*CD=(10^{i}A+10^{j}B)*(10^{i}C+10^{j}D)=10^{2i}AC+10^{2J}BD+$

$(10^{i+J}(A+C)B-10^{j+I}(B-D)A)$

$=10^{2i}AC+10^{2J}BD+(10^{i+J}BC+10^{j+I}DA)$

Or

$AB*CD=(10^{i}A+10^{j}B)*(10^{i}C+10^{j}D)=10^{2i}AC+10^{2J}BD+$

$(10^{i+J}(B+D)A-10^{j+I}(A-C)B)$

$=10^{2i}AC+10^{2J}BD+(10^{i+J}DA+10^{j+I}BC)$

I IS THE POWER OF 10 MULTIPLIED BY A,C

J IS THE POWER OF 10 MULTIPLIED BY B,D

A and C have the same power of 10 and B and D have the same power of 10

The above equation simplifies to

$10^{2J}AC+10^{2J}BD+(10^{I+J}AD+10^{I+J}BC)$

Which simplifies to

$10^{2J}(AC+BD)+10^{I+J}(AD+BC)$

Consequences

1. Square of numbers

$AB*CD=(10^{i}A+10^{j}B)*(10^{i}C+10^{j}D)=10^{2i}AC+10^{2J}BD+$

$(10^{i+J}(A+C)B-10^{j+I}(B-D)A)$,

Now for a square of two numbers, AB=CD, therefore we replace CD with AB.

$AB*AB=(10^{I}A+10^{J}B)*(10^{I}A+10^{J}B)=10^{2I}A^{2}+10^{2J}B^{2}+(10^{I+J}(A+A)B-10^{I+J}A(B-B))$

$=10^{2I}A^{2}+10^{2J}B^{2}+2(10^{I+J}AB)$

e.g

			5	6
*			5	6
	2	5	3	6
+		6	0	0
=	3	1	3	6

Table 5

I=1 , J=0		
$10^2(5^2)=2500$	$(10^0)6^2=36$	2500+36=2536
$2(10^{1+0}(5*6))=600$	2536+600=3136	

Table 6

$10^2(5^2)=2500(1)$	$(10^0)6^2=36(2)$	2500+36=2536 (3)
5*6=30(4)	30*2=600(5)	2536+600=3136(6)

Table 7

From the above computation compared to the old method of multiplication, we have reduced our steps of multiplication from 11steps to 6 steps. This saves a lot of processor's time and memory space, justifying the need of the new method.

2. Multiplication of numbers where the numbers in ones add up to 10 and the number in tens is common

$AB*CD=(10^iA+10^jB)*(10^iC+10^jD)=10^{2i}AC+10^{2J}BD+$
$(10^{i+J}(A+C)B-10^{j+I}(B-D)A)$

Now if the values in tens are common then we have $AB*AD=(10^iA+10^jB)*(10^iA+10^jD)= 10^{2i}(A^2)+10^{2j}(BD)+ (10^{I+J}(2A)B-10^{I+J}(B-D)A)$

$10^{2I}(A^2)+10^{2J}(BD)+ (10^{I+J}((2BA)-(AB-AD))$
$10^{2I}(A^2)+10^{2J}(BD)+ (10^{I+J}((BA)+(AD))$
$10^{2I}(A^2)+10^{2J}(BD)+ (10^{I+J}((B+D)A)$

			4	7
*			4	3
	1	6	2	1
+		4	0	0
=	2	0	2	1

Table 8

I=1, J=0		
$10^2(4*4)=1600$	$10^0(7*3)=21$	1600+21=1621
$10^{0+1}((7+3)*4))=400$		

Table 9

IF B+D=10
THEN, WE HAVE
$=10^{2I}(A^2)+10^{2J}(BD)+ (10^{I+J}((10)A)$
LET I=1 AND J=0, THEN, WE HAVE
$=10^2(A^2)+(BD)+ (10^2A)$
$=10^2(A^2)+ (10^2A)+ (BD)$
$= 10^2(1+A)A+BD$

This can be simplified to
$AB*AD=10^2 (A+1) A+ (BD)$
e.g

			4	7
*			4	3
	2	0	2	1

Table 10

I=1, J=0		
102(4*5)=2000	100(7*3)=21	2000+21=2021
A+1=4+1=5		
7*3=21(1)	4+1=5(2)	5*4=20(3)
2000+21=2021(4)		

Table 11

For cases were the sum of the one's value adds up to ten, and the ten's value is common, the number of steps of multiplication reduces to only four. This is less than half the number of steps of multiplication using the normal method which is 11.

3. When the numbers with the same powers of 10 are the same.

$AB*CD=(10^{I}A+10^{J}B)*(10^{I}C+10^{J}D)=10^{2I}AC+10^{2J}BD+$
$(10^{I+J}(A+C)B-10^{I+J}(B-D)A)$

B and D have the same power and therefore let them be the same

$AB*CB=(10^{I}A+10^{J}B)*(10^{I}C+10^{J}B)=10^{2I}AC+10^{2J}B^{2}+(10^{I+J}(A+C)B-10^{I+J}(B-B)A)$
$=10^{2I}AC+10^{2J}B^{2}+10^{I+J}(A+C)B$

e.g.

			9	4
*			7	4
	6	3	1	6
+		6	4	0
=	6	9	5	6

Table 12

I=1 , J=0		
$10^{2}(9*7)=6300$	$(10^{0})4^{2}=16$	6300+16=6316
$10^{1+0}((9+7)*4))=640$	6316+640=6956	

Table 13

9*7=63(1)	4*4=16(2)	6300+16=6316(3)
9+7=16(4)	16*4=64(5)	6316+640=6956(6)

Table 14

From the above the number of steps of multiply 94 and 74 is 6. Using the known method, the number of steps is 11. This reduces the numbers of steps by 5. This time is very significant to the computer processor

Or

$AB*AC=10^{2I}A^{2}+10^{2J}(B*D)+(10^{I+J}(B+D)A-10^{I+J}(A-A)B)$
$10^{2I}A^{2}+10^{2J}(BD)+(10^{I+J}(B+D)A)$

			3	6
*			3	8
	0	9	4	8
+		4	2	0
=	1	3	6	8

Table 15

I=1 , J=0		
$10^{2}(3*3)=900$	$(10^{0})6*8=48$	900+48=948
$10^{1+0}((6+8)*3))=420$	948+420=1368	

Table 16

4. Multiplication of numbers where all the numbers are different

$AB*CD=(10^{i}A+10^{j}B)*(10^{i}C+10^{j}D)=10^{2i}AC+10^{2J}BD+$
$(10^{i+J}(A+C)B-10^{j+I}(B-D)A)$
$=10^{2I}(AC)+10^{2J}(BD)+(10^{I+J}(BA+BC)-(AB-AD))=$
$10^{2I}(AC)+10^{2J}(BD)+(10^{I+J}(BC+AD))$

Or

$AB*CD=10^{2I}(AC)+10^{2J}(BD)+(10^{I+J}(B+D)A-10^{I+J}(A-C)B)=$
$10^{2I}(AC)+10^{2J}(BD)+(10^{I+J}(DA+CB)$

e.g.

			8	7
*			4	9
	3	2	6	3
+	1	0	0	0
=	4	2	6	3

Table 17

I=1,j=0		
10^2(8*4)=3200	10^0(7*9)=63	3200+63=3263
10^{0+1}(8*9+4*7)=1000	3263+1000=4263	

Table 18

4*8=32(1)	9*7=63(2)	3200+63=3263(3)
8*9=72(4)	7*4=28(5)	72+28=100(6)
3263+1000=4263(7)		

Table 19

The multiplication takes 7 steps. With the known method of multiplication, it took nine steps to multiply two digit values. This is two steps less. The is a very valuable time for the processor. It also saves on the memory space. It means that the processor has more time to perform other tasks.

Suppose that we wanted to multiply three numbers

AB*CD=10^{I+I}(AC)+10^{J+J}(BD)+ (10^{I+J}(A+C)B-10^{I+J}(B-D)A)
10^{2I}(AC)+10^{2J}(BD)+ (10^{I+J}(BA+BC)-(AB-AD))=
10^{2I}(AC)+10^{2J}(BD)+ (10^{I+J}(BC+AD))

					3	9	5
*					4	6	7
		1	2	5	4	3	5
+			5	4	0	0	0
				4	1	0	0
					9	3	0
		1	8	4	4	6	5

Table 20

5*7=35(1)	9*6=54(2)	3*4=12(3)
3*6=18(4)	4*9=36(5)	18+36=54(6)
3*7=21(7)	4*5=20(8)	21+20=41(9)
9*7=63(10)	6*5=30(11)	63+30=93(12)
125435+54000(13)	179435+4100(14)	183535+930(15)
=184465		

Table 21

We saw earlier that to multiply a three-digit number with another three-digit number would take us 22 steps. With the new method of multiplication, it takes 15 steps! This is seven steps less than the steps of the known method. This means that the bigger the number being multiplied the more we reduce the number of steps done compared to the known method. Since most of the times we are interested in multiplying huge values, it means adopting the new method will save a lot of time. This justifies the new invention.

5. Multiplication of a value with n numbers

Let n=4

AB*CD=10^{I+I}(AC)+10^{J+J}(BD)+ (10^{I+J}(A+C) B-10^{I+J}(B-D)A)
10^{2I}(AC)+10^{2J}(BD)+ (10^{I+J}(BA+BC)-(AB-AD))=
10^{2I}(AC)+10^{2J}(BD)+ (10^{I+J}(BC+AD))
Or
AB*CD=10^{2I}(AC)+10^{2J}(BD)+ (10^{I+J}(B+D)A-10^{I+J}(A-C)B)=
10^{2I}(AC)+10^{2J}(BD)+ (10^{I+J}(DA+CB)=
10^{2I}(AC)+10^{2J}(BD)+ (10^{I+J}(DA+CB))
e.g.

					2	3	1	6
					5	9	7	8
	1	0	2	7	0	7	4	8
		3	3	0	0	0	0	0
			1	9	0	0	0	0
				4	6	0	0	0
				3	0	0	0	0
					7	8	0	0
						5	0	0
+	1	3	8	4	5	0	4	8

Table 22

Conclusion

The new method can be applied for multiplication of any number. It will save time and also memory space for storing the intermediate results.

References

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